Selected Solutions for Chapter 23: Minimum Spanning Trees

Solution to Exercise 23.1-1

Theorem 23.1 shows this.

Let A be the empty set and S be any set containing u but not v.

Solution to Exercise 23.1-4

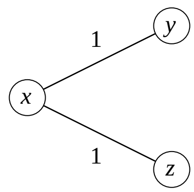
A triangle whose edge weights are all equal is a graph in which every edge is a light edge crossing some cut. But the triangle is cyclic, so it is not a minimum spanning tree.

Solution to Exercise 23.1-6

Suppose that for every cut of G, there is a unique light edge crossing the cut. Let us consider two minimum spanning trees, T and T', of G. We will show that every edge of T is also in T', which means that T and T' are the same tree and hence there is a unique minimum spanning tree.

Consider any edge $(u, v) \in T$. If we remove (u, v) from T, then T becomes disconnected, resulting in a cut $(S, V - S)$. The edge (u, v) is a light edge crossing the cut $(S, V - S)$ (by Exercise 23.1-3). Now consider the edge $(x, y) \in T'$ that crosses $(S, V - S)$. It, too, is a light edge crossing this cut. Since the light edge crossing $(S, V - S)$ is unique, the edges (u, v) and (x, y) are the same edge. Thus, $(u, v) \in T'$. Since we chose (u, v) arbitrarily, every edge in T is also in T'.

Here's a counterexample for the converse:



Here, the graph is its own minimum spanning tree, and so the minimum spanning tree is unique. Consider the cut $(\{x\}, \{y, z\})$. Both of the edges (x, y) and (x, z) are light edges crossing the cut, and they are both light edges.